Dynamic Mach cone as a diagnostic method in reactive dusty plasma experiments

O. Havnes,[1,*] T. W. Hartquist,[2] A. Brattli,[1] G. M. W. Kroesen,[3] and G. Morfill[4]

[1]*Department of Physics, University of Tromsø, N-9037 Tromsø, Norway*
[2]*Department of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[3]*Eindhoven University of Technology, NL-5600 MB Eindhoven, The Netherlands*
[4]*MPI für extraterrestrische Physik, D-85740 Garching, Germany*

We demonstrate how the observation of dynamic Mach cones in dusty plasma experiments allows one to follow the evolution of the dust acoustic wave velocity as charges on dust particles change, or as their mass changes. In experiments in which only the charge is changing due to, e.g., the application of a thin coating on UV-illuminated dust particles, the coating rate may be inferred through the analysis of the Mach cone pattern. In other experiments where the dust sizes are changing, also leading to some change in charge, the growth rate can be followed.

Havnes et al. [1,2] first suggested that Mach cones should be observable in dusty plasmas and serve as valuable diagnostics since they can be directly observed through remote sensing and they do not significantly affect the dusty plasma conditions. The Mach cones in dusty plasmas have since been observed in several experiments [3–5], have received further theoretical attention [6,7], and are now studied in many ongoing and planned experiments both on the ground and in space. The Mach cones, which occur if a localized body moves through a dusty plasma at a speed larger than the dust acoustic wave (DAW) [8–11] speed, are of a nature similar to those of cones occurring at supersonic speeds in gases, liquids, or some crystals [12,13].

Earlier investigations of Mach cone properties in dusty plasmas [1,2,6,7] have concentrated on cases in which the dust particles do not evolve. In the following we will consider cases of what we choose to call dynamic Mach cones (DMCs), where the evolution of the dust particle takes place. This can happen in reactive dusty plasmas by the growth of the particle sizes, which in turn will lead to a change in the charge. We also consider cases in which the charge changes, e.g., during a coating process. Systematic changes in the ionizing radiation or plasma conditions can also influence the Mach cone pattern. All these processes will lead the Mach cone pattern to deviate from a straight V-shaped pattern, which results for constant dust properties.

THE DAW AND TIME SCALES FOR DUST CHARGE CHANGES AND SIZE CHANGES

The DAW [8–11,14] can be subject to both Landau damping and to charge fluctuation damping [10,11]. We neglect these processes, they are generally not important in the dusty plasmas we will consider, and consider long-wavelength cases $k^2\lambda_D^2 \ll 1$ for which the DAWs are nondispersive. The dispersion relation then becomes [8,10]

$$\omega = kZ_d \left(\frac{n_{d0}}{n_{i0}}\right)^{1/2} \left(\frac{k_B T_i}{m_d}\right)^{1/2} \Big/ \left(1 + \frac{n_{e0} T_i}{n_{i0} T_e}\right)^{1/2}. \quad (1)$$

Here ω is the wave frequency and k is the wave number while n_{e0}, n_{i0}, and n_{d0} are the electron, ion, and dust number densities in the absence of a wave. The electron and ion temperatures are T_e and T_i, m_d is the dust mass, and Z_d is the dust charge number. Boltzmann's constant is k_B. The magnitude of the phase velocity (ω/k) of the DAW is low; it can be as low as a few cm/s or less in experiments, and down to a few m/s or less in planetary rings. The disturbing body creating the Mach cone, which could be a dust particle in experiments [3,4] or a boulder in planetary rings [1], must move at a speed $v_B > \omega/k$ for a Mach cone to be created. One issue for us to consider is whether the dust particles can change their properties during the lifetime of an observed Mach cone pattern, created during one particle passage through the experiment, or whether changes occur on a longer time scale so that successive Mach cone patterns show the evolution.

The time rate of the change of a dust particle's mass in reactive environments in which a specific gas, of number density n_g, temperature T_S, and mass m_g, accretes on to dust is given by

$$\frac{dm_d}{dt} = \pi r_d^2 n_g m_g v_g, \quad (2)$$

where

$$v_g = (8k_B T_S / \pi m_g)^{1/2}. \quad (3)$$

The time scale for the mass change is $t_{acc} = m_d/(dm_d/dt)$ given by

$$t_{acc} = \frac{4}{3} \frac{\rho_d r_d}{n_g m_g v_g}. \quad (4)$$

The time t_{coat} to obtain a coating of 1 ML of particles, if each coating atom or molecule occupies a surface of σ_{coat}, is given by

$$t_{coat} = \frac{4}{\sigma_{coat} n_{coat} v_{coat}}. \quad (5)$$

We consider parameter values typical for dust experiments of temperatures $T_g \sim 1000$ K, $m_g \approx m_{coat} \approx 30 m_H$ (leading to

*Electronic address: Ove.Havnes@phys.uit.no

$v_g = v_{\text{coat}} = 840$ m/s), $\rho_d = 3000$ kg/m^3, $r_d \sim 1$ μm $= 10^{-6}$ m, and $\sigma_{\text{coat}} = 10^{-19}$ m^2. We find for these values that

$$t_{\text{acc}} \sim \frac{9.5 \times 10^{19}}{n_g} \text{ s} \qquad (6)$$

and

$$t_{\text{coat}} \sim \frac{5 \times 10^{16}}{n_{\text{coat}}} \text{ s}. \qquad (7)$$

For laboratory conditions, the gas number density normally ranges upwards from 10^{18} m^{-3} and coating materials can be present at smaller or comparable number densities. The above time scales can, therefore, vary within very wide limits but can easily be made comparable to the propagation time for a DAW across the volume of an experiment. For today's dust experiments this is of the order of a few seconds but in some forthcoming larger experiments the time will be larger.

The charging time in rf-heated dust experiments is usually very short, sometimes as low as 10^{-6} s, so any charge changes are most likely almost instantaneous if changes in dust properties, plasma conditions, or charging illumination occur.

With the low densities in planetary rings of $n \gtrsim 10^6 - 10^7$ m^{-3} the time scales for accretion and coating will be much longer than the travel time for a DAW before it is damped. We will, therefore, in the following only consider the changes of the wave pattern of Mach cones in dust experiments.

EFFECT ON THE MACH CONE PATTERN BY CHANGES IN DUST MASS OR DUST CHARGE

In a nondispersive Mach cone the wave pattern basically has a pure V shape except possibly near the disturbing body [3,4,15]. If dispersion is important, this will not be so [6], but we will restrict attention to nondispersive cases. The V-shaped form results in the nondispersive regime because all of the disturbances propagate with the same speed α_{d0}, so that if a disturbing body is moving through the dusty plasma with speed v_B, the V pattern will have an opening half-angle γ given by

$$\sin \gamma = \frac{\alpha_{d0}}{v_B} \qquad (8)$$

e.g., [1,3,4]. Changes in the dust properties will influence the DAW properties and thereby the Mach cone wave pattern. Here we will consider two separate cases and show in each of them how the changes affect the Mach cone pattern.

If a body moves through a dusty plasma it will, at every point, create a disturbance, which propagates as a circle and contributes to a V-shaped pattern if the DAW speed is constant. If the DAW speed varies, the circle generated at a point χ (see Fig. 1), at time $t = -\chi/v_B$ will expand and have a radius at the time $t = 0$ of

$$R(\chi, t=0) = \int_{-\chi/v_B}^{0} \alpha_d(t) dt. \qquad (9)$$

We take $t = 0$ to be the time at which we observe, or obtain a picture of, the Mach cone. For both of the demonstration cases that we study, we assume that changes in dust mass and charge do not significantly affect [see Eq. (1)] n_{d0}, n_{i0}, n_{e0}, or the temperatures. This is equivalent to the dust charge density being small compared to n_{e0} and n_{i0}.

In case I we will consider dust that is illuminated by UV radiation and on which a coating process changes the photoelectric properties resulting in alterations of the dust charge also. The effect on the dust mass is taken to be negligible. We will assume that the dust charge number Z_d changes linearly as the coating proceeds and that

$$Z_d(t) = Z_{d0} + \dot{Z}_d t. \qquad (10)$$

Here Z_{d0} is the dust charge when we observe the Mach cone at $t = 0$. The charge number Z_d is the only quantity varying in Eq. (1). Inserting Eqs. (1) and (10) in Eq. (9) we find

$$R(\chi, t=0) = \frac{\chi}{M} \left(1 - \frac{1}{2} \frac{\chi}{t_q v_B} \right), \qquad (11)$$

where the charging time due to the coating is $t_q = Z_{d0}/\dot{Z}_d$ and $M = v_B/\alpha_d(0)$.

In case II only the mass changes due to gas accretion. The change in mass will affect the DAW directly through the $m_d^{-1/2}$ term but this will partly be compensated for by an increase in dust charge as the radius increases. We consider charging by plasma collisions and attachment. The surface potential of the dust is determined by the plasma temperature and the ion mass [16,17] and is independent of the dust radius. From this and the expression for the surface potential of a sphere, $U = q/4\pi\epsilon_0 r_d$, it follows that the change in charge number with dust radius is

$$Z_d(t) = Z_{d0} \left(\frac{r_d}{r_{d0}} \right). \qquad (12)$$

Inserting Eq. (12) and

$$m_d(t) = m_d(t=0) \left(\frac{r_d(t)}{r_{d0}} \right)^3 \qquad (13)$$

into Eq. (1) we find that

$$\alpha_d(t) = \alpha_{d0} \left(\frac{r_{d0}}{r_d(t)} \right)^{1/2}. \qquad (14)$$

With a constant density of accreting molecules, the dust radius will change linearly with time as

$$r_d = r_{d0} + \left(\frac{dr_d}{dt} \right) t \qquad (15)$$

so that with $t_r \equiv r_{d0}/(dr_d/dt)$ we have

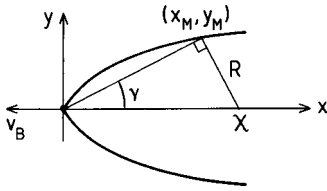$$\alpha(t) = \alpha_{d0} \left(1 + \frac{t}{t_r} \right)^{-1/2}. \qquad (16)$$

FIG. 1. The symbols used in calculating the Mach cone patterns for cases I and II in the text. The disturbance is caused by a body moving at a velocity $-v_B\mathbf{x}$, $x=\chi$ is the position of the body at time $t=-\chi/v_B$. γ is the half opening angle of the Mach cone. $R(\chi,t=0)$ is the radius to which the surface of influence generated by the body at $t=-\chi/v_B$ has grown by $t=0$. The curve coinciding with the Mach cone at $t=0$ may be considered to be given either as a function of x, $y=y_M(x)$, or as a function of y, $x=x_M(y)$.

Inserting Eq. (16) in Eq. (9) we get

$$R(\chi,t=0)=2t_r\alpha_{d0}\left[1-\left(1-\frac{\chi}{v_Bt_r}\right)^{1/2}\right].\quad(17)$$

A consideration of Fig. 1 allows us to relate the coordinates, x_M and y_M, of a point on the Mach cone to χ and $R(\chi,t=0)$,

$$x_M=\chi-R\sin\gamma=\frac{\chi^2-R^2}{\chi},\quad(18)$$

$$y_M=R\cos\gamma=\frac{R}{\chi}(\chi^2-R^2)^{1/2}.\quad(19)$$

We now use Eqs. (18) and (19) to plot the resulting DMC patterns for the two cases governed by Eqs. (11) and (17), respectively. For case I we use $M=2$, $v_B=4$ cm/s, and $t_q=1$ and 2 s in Eq. (11), which applies when only the charge changes. This corresponds to $\alpha_{d0}=2$ cm/s, which is close to the values found by Samsonov et al. [3,4]. We use the same parameter values for case II. The resulting DMC patterns are shown in Fig. 2. For case II, in which the size increase dominates, the DAWs slow down with time and the Mach cone pattern is concave. For a pure charge increase, as in case I, the DAW speed increases and the pattern becomes convex.
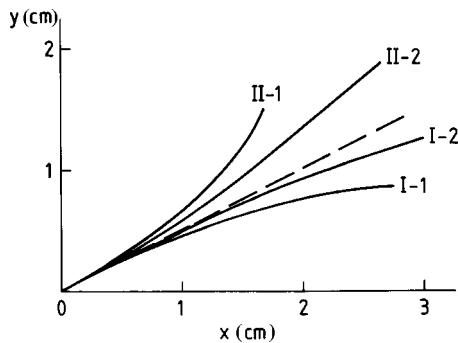

FIG. 2. The shapes of Mach cones. $y_M(x)$ is plotted for the upper branch of each of the five Mach cones. The straight line is for the case when the dust does not evolve, leading to α_d being constant. Results for cases I and II are shown for the choices of $M=2$ and $v_B=4$ cm s^{-1}. The second parameter in the label for each curve indicates the assumed value of t_r in seconds.

This demonstrates the potential of the DMC method to discriminate between different evolutionary effects on the dust.

EXTRACTION OF INFORMATION ON DUST CHANGES FROM OBSERVATIONS OF DYNAMIC MACH CONES

We assume that the observations allow a DMC pattern to be represented by a curve $y(x)$. We wish to extract the evolution of the DAW speed $\alpha_d(t)$ from $y(x)$.

From Fig. 1 we see that if we replace x_M, y_M with y, x,

$$\cos\gamma=\frac{x}{(x^2+y^2)^{1/2}}=\frac{(x^2+y^2)^{1/2}}{\chi},\quad(20)$$

which gives

$$\chi=\frac{x^2+y^2}{x}.\quad(21)$$

Furthermore,

$$\sin\gamma=\frac{R}{\chi}=\frac{y}{(x^2+y^2)^{1/2}}.\quad(22)$$

Inserting Eq. (21) in Eq. (22), we find

$$R=\frac{y}{x}(x^2+y^2)^{12}.\quad(23)$$

From the relationship between χ and t and from Eq. (21) we obtain

$$t=-\frac{\chi}{v_B}=-\frac{x^2+y^2}{xv_B}\quad(24)$$

and

$$dt=-\frac{1}{v_B}\left[\frac{x^2-y^2}{x^2}+\frac{2y}{x}\left(\frac{dy}{dx}\right)\right]dx.\quad(25)$$

Differentiation of Eq. (9) with respect to t, with R given by Eq. (23), and then differentiation with respect to x by the use of Eq. (25) allows the derivation of

$$\alpha_d(t)=v_B\left(\frac{-y^3+(x^3+2xy^2)\dfrac{dy}{dx}}{\left(x^2-y^2+2xy\dfrac{dy}{dx}\right)(x^2+y^2)^{1/2}}\right).\quad(26)$$

If we have a normal V-shaped Mach cone pattern where $y/x=\tan\gamma$, then Eq. (26) leads to the expected result of Eq. (8). Clearly from observations of a DMC pattern we can infer the change in the DAW velocity $\alpha_d(t)$ through the use of Eqs. (24) and (26). Knowing the variation of the DAW speed with time, we use Eq. (1) to find the change in charge if coating occurs, or the change in mass if accretion of significant amounts of material, similar in composition to that of the original grains, occurs.

DISCUSSION

We have shown that the DMC diagnosis method has the potential of providing essential information on reactive laboratory or industrial dusty plasmas in which the dust properties change.

Coating rates can now easily be made at ~ 1 nm/s, which can change the charge of the particle profoundly on the time scale of seconds. For industrial applications, deposition rates of up to ~ 1 μm/s have to be achieved. This will lead to very fast changes of dust properties, which the DMC method has the potential to monitor. Methods such as mass spectrometry, extraction and weighing, laser induced particle explosive evaporation, particle oscillation monitoring are not suited to determine the mass of the dust particles because they are invasive and disturb the plasma or destroy the particle. For determination of the charge, very few methods exist. Most of them are indirect and require model assumptions. The only direct method (photodetachment) requires a method to determine absolute values of the electron density, which for the density range under discussion here almost always necessitates specific discharge geometries. For time scales shorter than 1 s, no truly noninvasive and real time method for measuring mass and charge is available.

If the DMC method can be implemented in dusty plasmas used for particle or surface processing (silane plasmas used for solar cells, plasma based coating of dust particles), this would open up a perspective for real time process control and diagnosis of the processing of the particles. Even in the case of silane plasmas, where the particles are too small to visualize individually, the DMC method will work: the spatial particle density variations induced by the Mach cone can be studied without being able to ''see'' each individual particle, by using classical fluid dynamics diagnostic methods such as (Mach-Zehnder) interferometry and Schlieren photography. In the present two-dimensional (2D) imaging of position and velocity of dust particles affected by the DAW to form a Mach cone [4–6], the images are not very sharp and well defined. We expect that progress in imaging and image processing will lead to much improved knowledge of the 3D position and motion of dust particles, which should lead to a better definition of the Mach cone.

Nonuniformity within a plasma chamber will certainly affect the DMC profile, and the straightforward analysis presented here will, in such cases, not be valid throughout the chamber. Locally it may be applied. In any case, observations of deformed DMC should lead to valuable information on the localization of and degree of nonuniformity and as such be useful in a search for optimal chamber conditions.

We have not considered any size distribution of the dust particles. Brattli, Havnes, and Melandsø [18] found that the propagation velocity of the DAW could be represented by the expressions for a monosized dusty plasma if one uses average values for the sizes. However, little work has been done on DAW in dusty plasmas with a size distribution. Until a full theory for DAW is fully developed it may be difficult to draw full benefit in all natural situations from the DMC method to deduce plasma and dust conditions. The higher degree of control available in the laboratory favors the application of the method to experiments.

We have considered cases in which changes in dust properties affect an instantaneously observed Mach cone structure. This means that the DAW propagation time t_p for the disturbing body and the time scale for dust changes t_L are comparable. We can also consider cases in which t_L and t_p are not comparable. If $t_L \gg t_p$ one observation of the Mach cone will show a V-shaped pattern, but later observations will show the Mach cone to have a changed opening angle γ; use of Eq. (10) will give the time history of $\alpha_d(t)$. If $t_L \ll t_p$, which may happen in some etching experiments, where the particle surface chemistry may change in milliseconds, the change in the DMC may show up as a kink in the pattern. If the kink can be resolved Eqs. (24) and (26) can be used to determine the time history of α_d and related parameters. If not, the change of angle γ on each side of the kink will give the change in α_d, and the resolution will set an upper limit on the time for the change of dust properties.

---

[1] O. Havnes, T. Aslaksen, T. W. Hartquist, F. Li, F. Melandsø, G. E. Morfill, and T. Nitter, J. Geophys. Res. **100**, 1731 (1995).

[2] O. Havnes, F. Li, F. Melandsø, T. Aslaksen, T. W. Hartquist, G. E. Morfill, T. Nitter, and V. Tsytovich, J. Vac. Sci. Technol. A **14**, 525 (1996).

[3] D. Samsonov, J. Goree, Z. W. Ma, A. Bhattacharjee, H. M. Thomas, and G. E. Morfill, Phys. Rev. Lett. **83**, 3649 (1999).

[4] D. Samsonov, J. Goree, H. M. Thomas, and G. E. Morfill, Phys. Rev. E **61**, 5557 (2000).

[5] A. Melzer, S. Nunomura, D. Samsonov, and J. Goree, Phys. Rev. E **62**, 4162 (2000).

[6] D. H. E. Dubin, Phys. Plasmas **7**, 3895 (2000).

[7] O. Havnes, F. Li, T. W. Hartquist, T. Aslaksen, and A. Brattli, Planet. Space Sci. **49**, 223 (2001).

[8] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 4 (1990).

[9] M. Rosenberg, Planet. Space Sci. **41**, 229 (1993).

[10] F. Melandsø, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).

[11] F. Melandsø, T. Aslaksen, and O. Havnes, J. Geophys. Res. **98**, 13 315 (1993).

[12] H. W. Liepman and A. Roshko, *Elements of Gas Dynamics* (Wiley, New York, 1957).

[13] P. Gumbsch and H. Gao, Science **283**, 965 (1999).

[14] W. Pilipp, T. W. Hartquist, and G. E. Morfill, Astrophys. J. **387**, 364 (1992).

[15] A. Brattli, O. Havnes, and F. Melandsø, J. Plasma Phys. **9**, 3 (2002).

[16] L. Spitzer, *Physical Processes in the Interstellar Medium* (Wiley, New York, 1978).

[17] E. C. Whipple, Rep. Prog. Phys. **44**, 1197 (1985).

[18] A. Brattli, O. Havnes, and F. Melandsø, J. Plasma Phys. **58**, 691 (1997).